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Noise Radiation from the Side Edges of Flaps

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The recently observed phenomenon of high noise radiation from the side edges of flaps in flow is investigated by way of a simple two-dimensional model problem. The model is based upon a physical picture of boundary layer vorticity being swept around the edge by spanwise flow on the flap. The model problem is developed and solved and the resulting noise radiation calculated. Further, a mathematical condition for the vortex to be captured by the potential flow and swept around the edge is derived. The results show that the sound generation depends strongly upon the strength of the vorticity and distance from the edge and that it can be more intense than the more common trailing edge noise source in agreement with the experimental observations.

Introduction

TWO recent studies[1,2] of airframe noise radiation from complete wing systems have shown that a major noise source location for high flap deflection angles occurs at the side edges of part span trailing edge flaps. Thus, these side edges will be important sources of airframe noise during landing approach where the flap deflections are large. In addition, it now appears probable that the design of the next generation of aircraft, using present-day high-bypass-ratio engine technology, will require the flap side edges to actually intrude into the jet stream on landing approach. Thus, this phenomenon may prove to be an important new source of propulsive noise as well.

Although noise radiation from the trailing edges of wings and flaps has been identified and studied for some time (see Howe[3] for an excellent review), the experimental evidence seems to indicate that this new source can be more intense than trailing edge noise. Thus, an effort toward the understanding and control of this source is justified.

In this paper, the analysis of a simple model problem is presented which is believed to contain the basic physics of the phenomenon of interest. The model is based on this physical picture: Due to the flow into the flap, produced by the forward speed of the aircraft in the airframe noise case or the jet velocity in the propulsive case, a high pressure region is created on the underside of the flap. In an attempt to relieve this pressure, fluid spills around the side edge of the flap producing a flow along its span. This is the same process which gives rise to the familiar tip vortices on wings.[4] However, this spanwise flow must also satisfy the no-slip condition on the flap. Satisfaction of this condition will require the formation of positive vorticity in the chordwise direction on the underside of the flap which will then be swept around the side edge by the spanwise flow resulting in the radiation of noise.

The paper consists of four basic sections. In the first, the two-dimensional model problem involving a vortex in the presence of an infinite half plane with potential flow around its edge is developed and solved. In the second, a necessary and sufficient condition for the vortex to be swept around the edge is developed. This is due to the fact that the vortex under the influence of its image will actually move away from the edge. Thus, it will be swept around the edge only when the spanwise flow is strong enough to overcome the influence of the image. The third presents a calculation of the sound generated by the vortex motion using the low-frequency Green's function theory developed by Howe.[5] Finally, an example of the previously developed theory is included.

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Two-Dimensional Model Problem

Consider a vortex with positive circulation Γ initially below an infinite half plane in the presence of a flow as shown in Fig. 1a. Assuming the edge of the half plane corresponds to the flap side edge, this model problem will represent chordwise vorticity on the underside of the flap being swept around the flap side edge by the spanwise flow in regions away from the leading and trailing edges of the flap. Near these edges, three-dimensional effects would play a role but should not drastically change the physical processes displayed by the model. Away from the immediate vicinity of the flap side edge, the flow would, of course, be dominated by the flow into the flap (into the page of Fig. 1a). This flow would affect primarily only the propagation of sound produced by this model of the situation near the edge.

This same geometry in the absence of flow and with the vortex initially above the half plane has been considered by Howe[5] and Crighton[6] as an example of sound generation in the presence of surfaces. With the positive vortex initially above the half plane, the vortex will sweep around the edge under the influence of its image alone. The model has also been extended to the case of uniform flow in the positive x direction by Howe[7] and Obermeier[8] as a model of trailing edge noise. The unique feature of the present problem is the complex interaction of the image induced and potential flows which controls the gross behavior of the votex.

Introduce the complex variable $z=x+iy=Re^{i\theta}$ $(-\Pi<\theta\leq\Pi)$. Then, with the transformation $z=-\lambda^2$, the physical geometry may be transformed into that shown in Fig. 1b, where $\lambda=\zeta+i\eta$. If the flow is supposed to be uniform in the transformed plane, the complex potential may then immediately be written as

$$W(\lambda)=-A\lambda-\frac{i\Gamma}{2\pi}\ln(\lambda-\lambda_0)+\frac{i\Gamma}{2\pi}\ln(\lambda-\lambda_0^*) \qquad (1)$$

where $\lambda_0=\zeta_0+i\eta_0$ is the transformed vortex position, A controls the magnitude of the uniform flow in the negative ζ direction, and the asterisk indicates the complex conjugate. Since the complex potential is invariant under conformal mapping, the complex potential in the z plane is

$$W(z)=-Aiz^{1/2}-\frac{i\Gamma}{2\pi}\ln i(z^{1/2}-z_0^{1/2})+\frac{i\Gamma}{2\pi}\ln i(z^{1/2}+z_0^{*1/2})$$

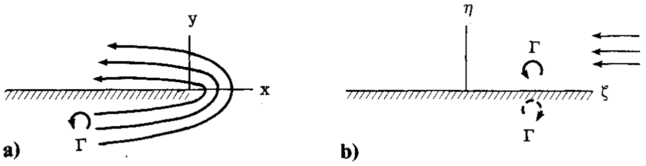

Fig. 1 Geometry of flow; a) physical plane, b) transformed plane.

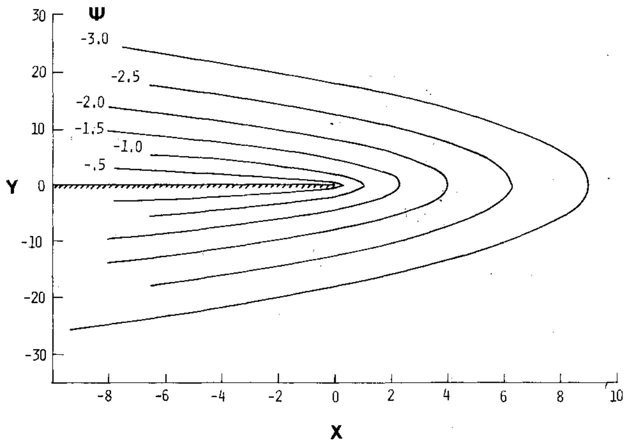

Fig. 2 The stream function $\psi(x,y)$.

where $z_0 = x_0 + iy_0$ is the vortex position. The velocity components in the physical plane are then

$$U - iV = \frac{dW}{dz} = \frac{-Ai}{2z^{1/2}} - \frac{\Gamma i}{4\pi z^{1/2}}\frac{1}{(z^{1/2} - z_0^{1/2})} + \frac{\Gamma i}{4\pi z^{1/2}}\frac{1}{(z^{1/2} + z_0^{*1/2})}$$

In the absence of the vortex, the flow in the physical plane is steady with streamlines as shown in Fig. 2. The velocity components fall off with distance from the edge with the upwash velocity along the line $y = 0$ given by

$$V_w = A/2x^{1/2} \qquad x > 0 \tag{2}$$

This flow is considered to be a reasonable representation of the flowfield in the immediate vicinity of the edge and away from the leading and trailing edges of the flap. See, for comparison, the edge flowfields measured by Francis and Kennedy,[4] which agree qualitatively with the assumed model flow from 20% of the chord on back to the trailing edge.

When the vortex is present, the flow becomes time dependent and, thus, capable of generating sound. The velocity of the vortex itself is determined most easily by Routh's rule.[9] It is

$$U - iV\Big|_{z=z_0} = \frac{-Ai}{2z_0^{1/2}} + \frac{i\Gamma}{4\pi(z_0 + |z_0|)} + \frac{i\Gamma}{8\pi z_0}$$

Letting $z_0 = R_0 e^{i\theta_0}$ leads to the equations

$$\dot{\theta}_0 = \frac{A}{2R_0^{3/2}}\cos\frac{\theta_0}{2} - \frac{\Gamma}{4\pi R_0^2} \tag{3}$$

and

$$\dot{R}_0 = \frac{A}{2R_0^{1/2}}\sin\frac{\theta_0}{2} - \frac{\Gamma}{8\pi R_0}\tan\frac{\theta_0}{2} \tag{4}$$

which govern the motion of the vortex.

In order to solve for the path of the vortex, introduce the variables

$$r = R_0^{1/2}\cos\theta_0/2 \qquad s = R_0\cos\theta_0/2 \tag{5}$$

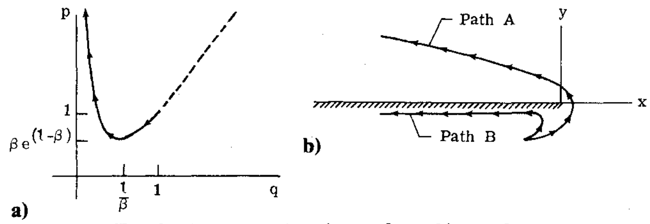

Fig. 3 Vortex paths; a) p-q plane, b) x-y plane.

Utilizing Eqs. (3) and (4), it can be shown that these variables are related by the simple equation

$$\dot{s}/s = (4\pi A/\Gamma)\dot{r} \tag{6}$$

Note that when $A = 0$, this yields the solution

$$s = R_0\cos\theta_0/2 = \text{const}$$

obtained by Crighton.[6] However, when $A \neq 0$, the solution for the path of the vortex is

$$s = s_0 \exp 4\pi A(r - r_0)/\Gamma \tag{7}$$

where

$$r_0 = R_0^{1/2}\cos\theta_0/2\,|_{t=0} \qquad s_0 = R_0\cos\theta_0/2\,|_{t=0}$$

This solution may be nondimensionalized and written in a more convenient form by letting

$$p = \frac{R_0^{1/2}}{R_0^{1/2}|_{t=0}} \qquad q = \frac{r}{r_0}$$

and noting that $(s/s_0) = pq$. Then, the solution becomes

$$p = (1/q)e^{\beta(q-1)} \tag{8}$$

where $\beta = (4\pi A r_0/\Gamma) \geq 0$. The case of primary interest here is for $\beta > 1$, i.e., the influence of the potential flow is initially more powerful than that of the image vortex. The vortex path in the p-q plane is as shown in Fig. 3a. The vortex always starts at the point $p = q = 1$ and traverses the path shown.

Back in the physical x-y plane, this path corresponds to two different types of behavior (for $\beta > 1$) as shown in Fig. 3b. On path A, the vortex leaves its starting position and is swept around the edge by the potential flow. On path B, the vortex is initially swept toward the edge by the potential flow, but as it comes nearer the half plane, the influence of its image becomes more powerful and it is ultimately swept away from the edge. In the next section, a criterion in terms of the parameter β is developed which differentiates between these two types of behavior.

Criterion for Circumvolution of Edge

A necessary and sufficient criterion for the vortex to be swept around the edge can be developed quite simply. From Eq. (8), it can be shown that the minimum value of p is $\beta e^{(1-\beta)}$ which occurs when $q = 1/\beta$ as shown in Fig. 3a. Now,

$$p = \frac{R_0^{1/2}}{R_0^{1/2}|_{t=0}}$$

Thus, R_0 always achieves a minimum, as seen in Fig. 3b. Further,

$$q/p = \frac{\cos\theta_0/2}{\cos\theta_0/2\,|_{t=0}} = \frac{e^{(\beta-1)}}{\beta^2} \tag{9}$$

when R_0 is minimum.

Now, if the vortex is swept around the edge, the minimum value of R_0 occurs when $\theta_0 = 0$ which implies from Eq. (9) that

$$e^{(\beta - 1)}/\beta^2 > 1 \tag{10}$$

as long as $\theta_0 |_{t=0} \neq 0$. On the other hand, if the vortex is not swept around the edge, the minimum value of R_0 occurs for $\theta_0 < \theta_0 |_{t=0}$ as can be seen in Fig. 3b. In this case, Eq. (9) implies that

$$(e^{(\beta - 1)}/\beta^2) < 1$$

Thus, the necessary and sufficient condition for the vortex to be swept around the edge is given by Eq. (10). For the case of interest, $(\beta > 1)$, this implies that

$$\beta = \frac{4\pi A R_0^{1/2} \cos\theta_0/2 |_{t=0}}{\Gamma} > 3.51286$$

Note that this condition is always satisfied as $\Gamma \rightarrow 0$. However, for finite values of Γ, the flow strength A must be sufficiently large.

Sound Radiation

The sound radiated by this vortex motion in the presence of the half plane can be calculated in several ways. Perhaps the simplest is to utilize the Green's function which satisfies the condition that its normal derivative on the surface vanishes. Then the sound generated may be computed entirely from a volume integral without the necessity for determination of the time-dependent pressure field on the surface. This Green's function has been obtained exactly by MacDonald[10] in the three-dimensional case and utilized by Ffowcs Williams and Hall[11] to analyze the trailing edge noise problem. However, this exact solution is in terms of Fresnel integrals with variable limits which involve the time dependent vortex position.

In order to simplify the problem somewhat, the "low-frequency" Green's function for this geometry obtained by Howe[5] in two dimensions will be employed. If the observer is at the position $x = (R\cos\theta, R\sin\theta)$ and the characteristic source frequency is sufficiently low, Howe showed that the acoustic pressure time history at the observer position could be calculated from the relation

$$p(x,t) \approx \frac{\rho_0 \Gamma \phi(x)}{2\pi^2 R} [\dot{x}_0 \cdot \nabla \psi] \tag{11}$$

where ρ_0 is the density of the ambient medium, $x_0 = (x_0, y_0)$,

$$\phi(x) = R^{1/2} \sin\theta/2$$

ψ is the streamfunction conjugate to ϕ which is exactly that shown in Fig. 2, i.e.,

$$\psi(x_0, y_0) = -R_0^{1/2} \cos\theta_0/2 \tag{12}$$

and the bracketed term is evaluated at the retarded time $t - R/c$ where c is the speed of sound in the ambient medium. The sound generation is seen to be proportional to the rate at which the vortex cuts through the streamlines of Fig. 2. Although Howe utilized this relation for a particular vortex path, it is, in fact, valid for arbitrary (unforced) vortex motion.

Now, the vortex velocities are given by Eqs. (3) and (4). Utilizing these with Eq. (12) leads to the relation

$$\dot{x}_0 \cdot \nabla \psi = -(\Gamma / 16\pi R_0^{3/2}) \sin\theta_0/2$$

Therefore,

$$p(x,t) \approx -\frac{\rho_0 \Gamma^2}{32\pi^3 R^{1/2}} \left[\frac{\sin\theta_0/2}{R_0^{3/2}}\right] \sin\theta/2 \tag{13}$$

yields the time history of the acoustic pressure at the observer position. Note that there is no explicit dependence upon the flow strength parameter A. However, there is a strong inverse dependence upon R_0, the distance of the vortex from the edge. For a given starting position, the minimum value of this distance will tend to be much less in the present case where the vortex is swept around the edge than it would be in the trailing edge case where the vortex is merely swept past the edge. This fact would appear to account for the high noise source seen at the side edge of flaps in the experimental data.

Another possible reason for the higher intensity of the side edge noise source can be seen by actually working out the details of the trailing edge case. This solution may be obtained in a similar fashion to the present analysis by merely replacing the first term in Eq. (1) by $-U_0 \lambda^2$ where U_0 is the magnitude of the uniform velocity in the positive x direction in the physical plane. For this case, the material derivative of the stream function becomes

$$\dot{x}_0 \cdot \nabla \psi = -\frac{U_0}{2R_0^{1/2}} \cos\theta_0/2 - \frac{\Gamma}{16\pi R_0^{3/2}} \sin\theta_0/2$$

where a term dependent upon the flow velocity now appears. For the case of interest, $\Gamma > 0$ and $\theta_0 < 0$ (or equivalently, $\Gamma < 0$ and $\theta_0 > 0$), these two terms are of opposite sign and thus will tend to cancel each other.

Of course, in order to utilize this theory to actually calculate the sound radiated from a flap side edge, it would be necessary to know, at least in a statistical sense, the strengths, initial positions, and formation times for all vortices swept around the edge and then superimpose the individual solutions to obtain the resulting total acoustic field. However, this model problem consisting of a single vortex being swept about the edge should illustrate the basic physics of the phenomenon.

Example of Theory

As an example of the previously developed theory, Fig. 4a shows the paths in the x-y plane taken by the vortices for different values of β. These paths were obtained by numerical integration of Eqs. (3) and (4). Distances have been nondimensionalized by

$$\delta = |R_0 \sin\theta_0 |_{t=0}|$$

the initial normal distance of the vortex from the half plane while velocities have been nondimensionalized by Γ/δ. The two types of behavior as a function of β as discussed in the previous section are illustrated. Note particularly the exceedingly close approach of the vortices to the edge. These paths may be contrasted with those calculated by Obermeier[8] for the trailing edge case where the vortices remain much further from the edge.

In Fig. 4b, the corresponding sound generation is depicted. These time histories were calculated from Eq. (13). The acoustic pressure value plotted has been nondimensionalized and normalized. It is given by

$$p_a(R,\theta,t) = \frac{32\pi^3 (R/\delta)^{1/2} p(x,t)}{\rho_0 (\Gamma/\delta)^2 \sin\theta/2}$$

In addition, the acoustic delay time R/c has been neglected. Note that when the vortex is not swept around the edge ($\beta = 2.86$), there is some low frequency sound radiation which occurs near the point where the vortex is being captured by its

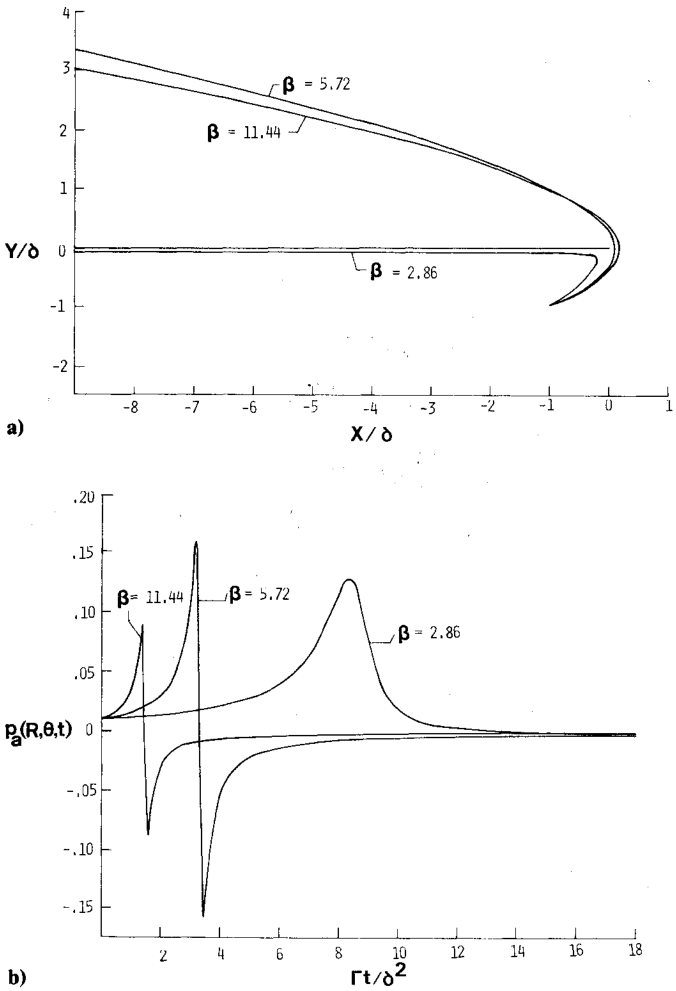

Fig. 4 a) Computed vortex paths in x-y plane; b) acoustic pressure time histories.

image to be swept back along the plane. However, when the vortex goes around the edge, much higher frequency sound is radiated with a zero occurring at the time the vortex crosses the x axis. The frequency of the radiated sound appears to increase monotonically with β due to the higher accelerations which occur. However, the amplitude is higher for $\beta = 5.72$ than it is for $\beta = 11.44$, apparently due to the vortex approaching closer to the edge (see Fig. 4a).

Conclusions

In this paper, the phenomenon of sound radiation from the side edges of flaps has been investigated in terms of a simple two-dimensional model problem. The model was developed to simulate the physical picture of vorticity generated on the underside of the flap being swept around the side edge by the spanwise flow. Insofar as the model contains the basic physics of the process of interest, the following conclusions may be drawn:

1) The magnitude of the sound radiation depends strongly upon the strength of the vorticity swept around the edge as well as the distance of the vortex from the edge. This result suggests that the various modifications which have been employed on wings to reduce the tip vortex might also be utilized on flaps to change the path of the vortex with respect to the edge and thus reduce the radiated sound.

2) For a given vortex strength and initial position, sound generated by the side edge may be larger than that generated by the trailing edge due to the fact that the vortex passes much closer to the edge in the former case. This result is in agreement with the experimental observations.

3) The phenomenon of boundary layer vorticity being swept around an edge involves a complex competition between the image induced and potential flows for which a criterion has been developed. Although this condition is apparently generally satisfied and the magnitude of the flow around the edge is largely controlled by the lift distribution on the flap, this result suggests that it might be possible to reduce the noise radiation by so modifying the edge flow that very little vorticity is captured and swept around the edge.

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